Problem 1.14

The voltage $v(t)$ across a device and the current $i(t)$ through it are

$$v(t) = 20\sin(4t)\text{ V and } i(t) = 10(1+e^{-2t})\text{ mA}$$

Calculate:

(a) the total charge in the device at $t = 1$ *s*, $q(0) = 0$.

(b) the power consumed by the device at $t = 1$ s.

[This 's' should not be italicized.]

Solution

Part (a)

Begin with the basic definition of current and integrate both sides with respect to time from 0 to 1.

$$\frac{dq}{dt} = i(t)$$

$$\int_0^1 \frac{dq}{dt}\,dt = \int_0^1 i(t)\,dt$$

$$q(1) - q(0) = \int_0^1 10(1+e^{-2t})\,dt\text{ mA}$$

$$q(1) - 0 = 10\int_0^1 (1+e^{-2t})\,dt\text{ mA}$$

Therefore, the charge in the device at $t = 1$ is

$$\begin{aligned} q(1) &= 10\left(t - \frac{1}{2}e^{-2t}\right)\Big|_0^1\text{ mC} \\ &= 10\left(1 - \frac{1}{2}e^{-2} + \frac{1}{2}\right)\text{mC} \\ &\approx 14.3\text{ mC}. \end{aligned}$$

Part (b)

The power that the device consumes is

$$p(t) = v(t)i(t) = [20\sin(4t)\text{ V}][10(1+e^{-2t})\text{ mA}] = 200(1+e^{-2t})\sin 4t\text{ mW}.$$

At $t = 1$ in particular,

$$p(1) = 200(1+e^{-2})\sin 4\text{ mW} \approx -172\text{ mW}.$$